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swered. For objective reality—fortune, material goods, institutions, offices and humanity and God—stands over against the finite self, not as some absolute "other," but as its own potential self. The Absolute, that is to say, is my real and complete self.

I may point the matter familiarly in this way. When we read in the Scriptures that "God so loved the world," habitually we not only misplace the emphasis but also suppose that this act of the deity was wholly gratuitous and gracious on his part. But from the very nature of consciousness as social, anything less than the complete inclusion of the world (i. e., the totality of human spirits) in God's love was impossible. And so must it be in our own case. Anything less than the identification of our finite, actual self with the Absolute, who is our infinite potential self, is logically impossible and morally futile.

To sum up: When the self-realizationists charge the rationalists with reducing morality to formalism, we may justly reply that their own concept of personality is a pure *a priori* product and their "idea of self" an empty abstraction. Their apriorism confines attention too much to the subjective content and meaning of personality, and their maxim, "Realize self" compels us again to ask the traditional question, "*Which* self?" And thus we never get beyond the problem of the relation of sensibility and reason in the life of the individual as such. On the other hand, the inquiry, "Who (or What) is my self?" has a direct empirical answer in the social nature of consciousness. On this datum of psychology as a stepping stone, we may rise without fear of hindrance or contradiction into the metaphysical zone. To be sure, psychology has nothing to say as to whether the universe is a personality or not. But for ourselves, assuming the proof of spiritual monism, "Realize self" becomes a concrete, practical maxim. For although we must wait on experience and social evolution for the knowledge of the means of self-realization, we are always sure of the nature of self and the content of the moral ideal. "Realize self" now means, "In your own finite life fulfil and perfect the life of the Absolute Spirit."

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A TWENTIETH CENTURY ZENO.

In considering any attempt to prove Euclid's parallel postulate, it is well to consider first what is meant by geometrical proof. In defining some terms by means of others, it is obvious that we must begin primarily with some terms which are themselves *left undefined.* To *prove* any statement is to show that it follows as a logical consequence from relations already accepted, so that we must eventually begin likewise with statements for which *no proof is offered.* These initial statements are frequently called "axioms" but had better be called "assumptions," since we are at liberty to make choice of these statements in any way we please, subject to but one condition, namely, that they must not contradict one another. For an ideal system they must be likewise independent—that is, such that no assumption follows as a logical necessity from one or more of the others.

It will be apparent, therefore, that our ability to prove any theorem will depend upon our initial assumptions, and that without a knowledge of the assumptions employed, the proving of a statement is meaningless. Now from very early times it was thought that Euclid's parallel postulate could be deduced from his other assumptions, or, in other words, that his system was redundant; and the different places in the ancient editions occupied by this historic statement would seem to indicate that Euclid himself was somewhat in doubt as to the necessity of inserting it. Thus have arisen the attempts to prove this assumption—that is, to deduce it from his other assumptions; and though all these attempts have failed, they have nevertheless been most fruitful in illuminating the subject of the foundations of geometry. By making *some other assumption* we may easily prove Euclid's; for example, it is now customary to use some such modification of Playfair's assumption as "Through a point without a line there cannot be two parallels to the line." Such substitution, however, is not contemplated in an attempt to prove the parallel postulate.

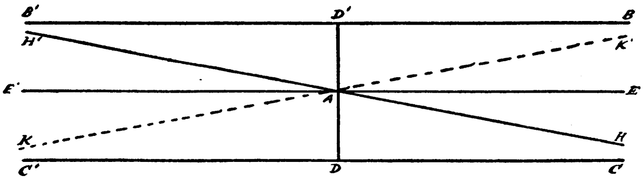
Conversely, to *disprove* a statement, is to show that it contradicts, or leads to a contradiction of, some other statement logically deduced from the original assumptions. That one discredits a statement is no more a logical objection to it than is one's credence to be regarded as logical proof. One is at liberty to *replace* any assumption of an ideal system by any other likewise independent of the remaining assumptions and evolve a geometry differing materially from the former, the sole logical criterion being that of consistency; but one is *not* at liberty to *add* assumptions, not contemplated by the author of a proposed system, nor can the latter be held responsible for contradictions or inconsistencies in consequence of the liberty so taken. Now the geometry of Lobatchevsky consists in *replacing* Euclid's parallel postulate by an assumption which we may express thus: "Through a point not on a given line we can have more than one line not intersecting the given line," the system being, of course, co-planar. Any logical objection to the resulting system must not therefore assume the Euclidean postulate directly or indirectly by employing some theorem which is based on this postulate.

These considerations are evidently overlooked by the writer of "A Modern Zeno" in the April number of *The Monist.* For example, he says (page 294):

> "Now a right-angled isosceles triangle may be dissected into two other half size right-angled isosceles triangles by a line drawn from the mid-point of the hypotenuse to the vertex of the right angle...."

If the author can prove this statement without using Euclid's parallel postulate, or one equivalent to it, his demonstration will be by no means "an imputation upon the reader." If, as we are certain will prove to be the case, he finds it necessary to use this postulate, then he has *added* it unlawfully to the system of Lobatchevsky and is himself responsible for all resulting incongruities.

On page 301 he considers the following figure, where BB′ and CC′ are perpendicular to DD′, A being the mid-point of DD′ and AH a parallel to DC through A:



He shows (as is easily seen by symmetry) that AH′ || D′B′, and then shows apparently that

B′D′B || H′AH,
H′AH || C′DC,
and therefore B′D′B || C′DC.

He seems to overlook the fact that in the geometry of Lobatchevsky parallelism is a *sensed relation,* a fact apparent enough, however, in the author's own quotations, as for example,

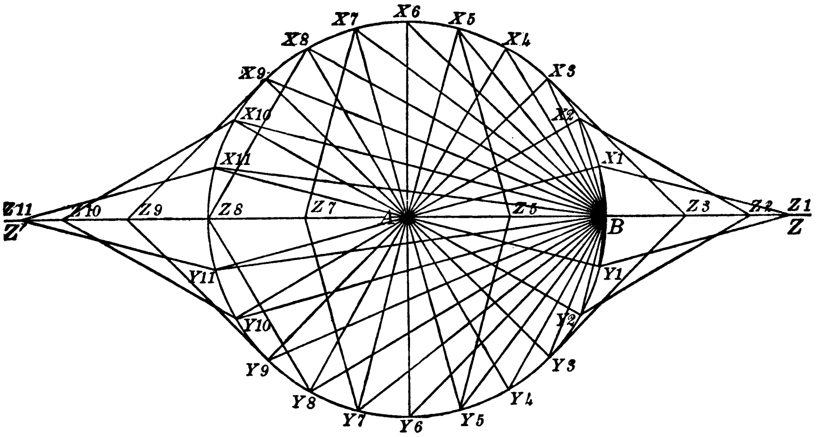
> "We must allow two parallels, one on the one and one on the other side" (page 292).
>
> "Under this assumption we must also make a distinction of *sides in parallelism*" (page 292).

"The farther parallel lines are prolonged on the side of their parallelism, the more they approach one another" (page 294).

In other words, if AH || DC, then AH is *not* parallel to CD. By keeping this in mind one can easily discover the fallacy of his course of deduction "no step of which is unsanctioned by the 'system' of Lobatchevsky" (!) by which he proves that in the above figure KAK′ is parallel to HAH′. These, among many other fallacies to which attention might be called, show that the author's criticism of the system of Lobatchevsky is not very formidable.

The author appears to be likewise unfortunate in his constructive work. In proposing a definition for a straight line, he says (pp. 304 and 305):

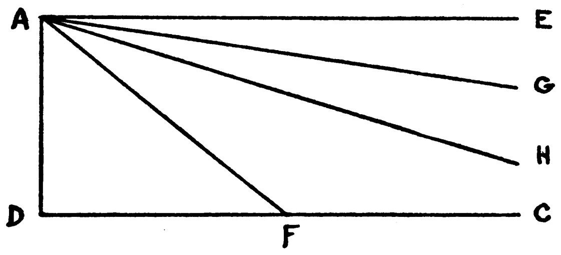
"Take any two points, say A and B. With, say, A as a turn-point (it might just as well have been B) and with the interval AB as the compass opening, scribe the circle BX_1X_2, etc. clear around complete. Then with B as turn-point and with *any* opening of the compass, short of 2AB, mark off on the first circle two points, say X_1 and Y_1. The same will be, of course, at equal intervals from B. Then from each of the points so marked scribe circles with the compass opening the interval AB. Such circles will all pass through A, but besides that they will otherwise intersect and determine a point as, say Z_1" (Z probably stands for Zeno).



Now what is AB? A straight line segment (or *sect,* to use Dr. Halsted's happy term)? If it be a sect, then he is using a sect in defining the property of straightness! If it be not a sect, then what significance has 2AB? In speaking of the points X, Y, Z, he assumes (apparently unconsciously) that under certain conditions which he does not explicitly set forth, two circles may have two common points, but no more. The condition under which they have only one common point is involved in his construction by circles centered at X_6 and Y_6, yet logically he cannot consider this important exception to his own statements. Again, in assuming that two circles cannot have three common points he is virtually assuming Euclid's parallel postulate with a few theorems included. The conditions under which they have two common points require for their consideration the definition of points within and without a circle, which definition requires a comparison of *sects,* the conditions involving in their ultimate analysis the assumption that a *straight line* through a point *within* a circle has one point (and therefore two points) in common with the circle! Also, the transition from an aggregate of distinct points (and these ununiformly distributed) to that of a continuous line is, if not impossible, at least quite difficult.

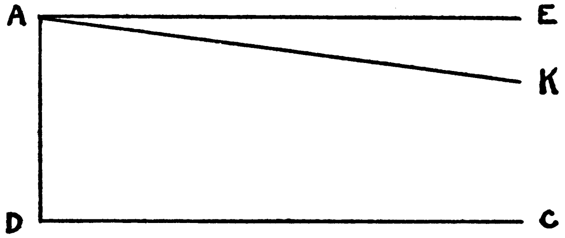
Another point which should be considered is that of simplicity in the initial assumptions; just as chemistry is founded on elements and biology begins with single cells, so geometry should have for its basis the simplest possible assumptions, each consisting, so far as possible, of a single statement. It was in all probability, the complexity of Euclid's parallel postulate, "If a Right Line, falling upon two other Right Lines, makes the inward Angles on the same side thereof, both together, less than two Right Angles, those two Right Lines, infinitely produced, will meet each other on that Side where the Angles are less than Right ones," which maintained interest in it, even if it did not afford the initial grounds for suspicion, rather than with regard to the relatively simple assumption which usually immediately precedes it, "Two Right Lines do not contain a Space." This simplicity which is in accord with the treatment of geometry by all modern critics is at variance with the course pursued by the author of the article in question.

Again, referring to AH as the boundary of lines from A cutting DC, as AF, and not cutting DC, as AG, he says (page 297),



"But he definitely puts his parallel among the lines that do not cut. But how about the relation of that parallel to the next line, that is, the last of the lines that cut DC? Does it make an angle with the parallel or is it the same line?"

Consider the same argument in connection with the following figure in orthodox Euclidean geometry, where AE and DC are perpendicular to AD and are therefore parallel. All other lines (or *rays*) in the angle DAE intersect DC. Let AK be "the last of the lines that cut DC. Does it make an angle with the parallel or is it the same line?....



"If the lines make an angle I suppose that that angle can be bisected, indeed *n*-sected, and such section-lines will be lines that neither cut nor non-cut. If the lines are only one single line then we have a line that both cuts and non-cuts."

Of course this argument is entirely fallacious, but it applies equally badly, nevertheless, to the geometry of Euclid and to Lobatchevsky. Other portions of the article are, of course, equally vulnerable.

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Mr. F. C. RUSSELL STILL DEMURS.

To the Editor of The Monist:

I wish to thank the Editor for his considerate notice of my article "A Modern Zeno" in *The Monist* of April, 1909. I think, however, that he is in error as to my assumption of the straight line. It was my special and paramount solicitude to avoid that assumption, and it seems to me that I have succeeded. But a discussion of the points involved would make this reply too prolix.

I intended to make, and I thought I made, my article a distinct *plea for better information.* I judged myself an example of a numerous class who seem to themselves to have good geometrical faculty, and who are warranted in that persuasion by a body of confirmations independent of their own esteem, and yet who are per-